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DEPARTMENT OF COMMERCE
BUREAU OF STANDARDS
WASHINGTON

Letter Circular LC 313

(December 21, 1931)



# NOMOGRAPHIC CHART FOR USE WITH STEEL MEASURING TAPES

## I. Introduction and Mathematical Formulae

The standard test for a 100-foot steel tape at the National Bureau of Standards is made with the tape supported horizontally throughout its length at room temperature under a tension of 10 pounds (for tapes over 100 feet in length a tension of 20 pounds is used). From the length thus observed, the length at 68°F. (20°C.) is computed, assuming a coefficient of thermal expansion of 0.000 006 45 per 1°F. (0.000 011 6 per 1°C.). Other tests are made when requested.

Tapes may be used in the field either under a tension at which they have been standardized in the laboratory, or under the tension which will give the nominal length of the tape, i.e., the tension of accuracy. In the first case corrections must nearly always be made on account of changes from the standard temperature or of changes in the method of support; in the second case no corrections need be made. Using the usual methods of computation it is not easy to calculate the tension of accuracy of a tape at a given temperature when supported at equidistant points. An attempt to remedy this condition is the basis of this discussion.

The length of a tape supported at equidistant points at the same height and held under tension is given with sufficient accuracy by the formula

$$L = L' \left[ 1 + \frac{P}{AE} + \alpha (t - t_o) - \frac{1}{24} \left( \frac{wd}{P} \right)^2 \right] \qquad (1)$$

where

$L'$ = distance between the terminal graduations when the tape is lying flat under no tension at the standard temperature. This is actually never realized,

$L$ = distance between terminal graduations on tape under the given conditions,

$P$ = tension on tape,

$A$ = cross-sectional area of tape,

$E$ = Young's Modulus of Elasticity of tape,

$\alpha$ = coefficient of thermal expansion of tape,

$t$ = temperature,

$t_o$ = standard temperature,

$w$ = weight of tape per unit length,

$d$ = span or distance between equidistant points of support.

These should be in consistent units: for example all lengths in inches, weights and tensions in pounds, Young's Modulus in pounds per square inch, temperatures in degrees Fahrenheit, and expansion in inches per inch per 1° Fahrenheit.

Here $\frac{L'P}{AE}$ is the stretch under tension, $L'\alpha(t - t_o)$ the elongation under the temperature change, and $\frac{L'}{24}\left(\frac{wd}{P}\right)^2$ the sag correction calculated under the assumption that a parabola is a sufficiently close approximation to the actual catenary.[1]

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[1] For equation (1) see any standard textbook on surveying, or the following references on the theory of metallic tapes:

Woodward, R.S., On the Measurement of the Base Lines at Holton, Ind., and at St. Albans, W. Va., App. 8, U.S. Coast & Geodetic Survey Report for 1892.

Woodward, R.S., Mathematical Theory of Metallic Tapes, U.S. Coast & Geodetic Survey Report for 1892, App. 8, pp. 480 to 489.

Jaderin, E., On the Measurement of Base Lines with Steel Tapes and with Steel and Brass Wires. (Translated by J. H. Gore), U.S. Coast & Geodetic Survey Report for 1893, App. 5.

Henrici, Prof. O., and Henrici, Capt. E. O., Theory of Tapes in Catenary. Professional Papers No. 1 (New Series), Ordnance Survey of Great Britain, 1912.

Young, A. E., On the Form of a Suspended Wire or Tape, including the Effect of Stiffness, Phil. Mag. 29, p. 96; 1915.

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Since all these corrections are small they may be treated as independently additive. Designating the condition of calibration by the subscript ($_o$) and the field conditions by the

subscript ($_1$) the tension of accuracy is determined by the equation $L_1 = L_o$ = nominal length of the tape.

$$L' \left[ 1 + \frac{P_1}{AE} + \alpha (t_1 - t_o) - \frac{1}{24} \left(\frac{wd}{P_1}\right)^2 \right] = L' \left[ 1 + \frac{P_o}{AE} \right] \tag{2}$$

where

$P_o$ is the tension of accuracy of the tape at standard temperature $t_o$ when supported throughout

and

$P_1$ is the tension of accuracy of the tape at temperature $t_1$ when supported at equidistant points at a distance d apart.

After dividing by $L_1$ and rearranging one obtains the cubic equation

$$P_1^3 - P_x P_1^2 - K = 0 \tag{3}$$

where

$$K = \frac{AE\ w^2 d^2}{24}$$

and

$$P_x = P_o - AE\alpha(t_1 - t_o)$$

(Here $P_x$ is the tension of accuracy of the tape when supported throughout at temperature $t_1$)

This equation (3) may readily be solved for the tension of accuracy $P_1$ by any of the usual methods for the numerical solution of algebraic equations but may more readily be handled in this case by a nomographic chart, as suggested to the author by Dr. L. B. Tuckerman.

The equation is in Soreau's[2] canonical form $IV_o$ permitting

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[2] Soreau, Nomographie, pp. 172, 174, 184 and 192 Chiron, Paris, 1921.

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parallel linear scales of $\frac{AE\ w^2 d^2}{24}$ and $P_o - AE\alpha(t_1 - t_o)$ with a curved functional scale of $P_1$. The multiplication of

$AE\alpha$ by $(t_1 - t_o)$ permits of two parallel linear scales of $AE\alpha$ and $AE\alpha(t_1 - t_o)$ with an inclined projective scale of $(t_1 - t_o)$. The subtraction of $AE\alpha(t_1 - t_o)$ from $P_o$ permits of three parallel linear scales of $AE\alpha(t_1 - t_o)$, $P_o$, and $P_o - AE\alpha(t_1 - t_o)$. Since both the scales of $AE\alpha(t_1 - t_o)$ and $P_o - AE\alpha(t_1 - t_o)$ are linear the three nomograms may readily be combined in one as shown in Figure 1.

## II. Experimental Work

### a. Verification of the Tension of Accuracy Formula.

Experimental determinations of the tension of accuracy were made using a group of 45 tapes, 100 feet in length, which were being studied for some of their physical characteristics. The tension of accuracy was determined experimentally for each of the 45 tapes, when supported at the 0 and 100-foot points, at four or more temperatures between 50° and 95°F. In making these observations special precautions were taken to obtain an accuracy of 0.1 pound. The observed values of tension of accuracy were found to agree with the computed values to within 0.1 to 0.3 pound.

### b. Constants to be Used in the Formula.

#### 1. Coefficient of thermal expansion.

The coefficient of thermal expansion of steel tapes has been assumed at the National Bureau of Standards to be 0.000 006 45 per degree Fahrenheit (0.000 011 6 per degree Centigrade), this being the best value obtained by experiment, and one from which the variations have been found to be practically negligible. This value was checked in this investigation as being applicable to these 45 tapes, some of which were made of ordinary tape steel, and others of stainless steel. The group of tapes having the maximum deviation from the value given above has coefficients so near this adopted value that an error of less than 0.0005 foot in 100 feet for a change in temperature of 30°F. is made if this nominal value is used instead of the experimentally determined value.

#### 2. Young's Modulus of Elasticity

Young's Modulus of elasticity may be assumed as $28.0 \times 10^6$ pounds per square inch. This is the mean value, experimentally determined in this investigation, and is in agreement with the results of other tests of tapes at the Bureau of Standards. This mean value may for all practical purposes be used for any steel

tape because the experiments just referred to indicate that the maximum error introduced by this assumption in using a 100-foot tape with a change in tension of 15 pounds would only be about 0.0005 foot.

Because of the difficulty in obtaining a sufficiently precise value for the cross-sectional area in many cases, it is often preferable when using this chart to determine the value for the product AE by measurements of change of length made on the tape when supported throughout at constant temperature at two tensions differing by 15 pounds or more, since AE is equal to $\frac{(P_2 - P_1)\ L}{\triangle L}$ where $\triangle L$ is the change in the length L when the tension is changed from $P_1$ to $P_2$.

### 3. Correction for Sag

The sag correction was checked experimentally and found to be correct for the purpose of this "tension of accuracy" formula to better than 0.001 foot.

## III. Nomographic Chart

A nomographic chart was constructed which was applicable to any of the 45 tapes tested in the above research. Nearly all of the types and sizes of 100-foot steel tapes manufactured and used in the United States were represented in this research. The chart is applicable to tapes of any length within the range of the scales of the nomogram.

Other nomograms may be drawn for tapes whose constants do not come within the scales on this nomogram.

Referring to the nomogram, Figure 1, the values for the 45 tapes of $AE\alpha$, axis "A", varied from 0.3 to 0.9. The inclined projective scale of $(t_1 - t_o)$, "B", is calibrated in terms of the observed temperature, $t_1$, from 30 to 110°F. The numerical value of the product $AE\alpha$ and $(t_1 - t_o)$ is not necessary, and axis "C" is a dummy axis. While the value, $P_o$, axis "D", is generally positive, it is made for the sake of uniformity in units from -40 to 40 pounds. (For purposes of computation the value of $P_o$ may theoretically be negative for some tapes.) Axis "E" is a dummy axis, but in case the value $P_x$ is known at the observed temperature, $t_1$, it is calibrated in pounds from -40 to 40. Axis "G" is calibrated in units between 0 and 20,000, and the tension of accuracy curve "F" is given in units between 4 and 40 pounds, which tensions are the approximate limiting usable values for 100-foot tapes.

The terms used in the nomogram should be stated in the following consistent units:-

A = Young's modulus of elasticity in pounds per sq. in.

E = cross-sectional area of tape in sq. in.

Note: It is advised that AE be determined from

$$\frac{(P_2 - P_1)\,L}{\Delta L}$$ (See Sect. IIb - 2, page 4)

$\alpha$ = coefficient of thermal expansion (.00000645 per °F.)

$t_o$ = standard temperature, 68°F.
$t_1$ = observed temperature in °F.
w = weight of tape in pounds per foot

d = distance between equidistant points of support in feet.

$P_o$ = tension in pounds at which tape has its nominal length when supported throughout at standard temperature.

$P_1$ = tension in pounds to be applied to tape when used at the observed temperature, $t_1$, and supported at equidistant points at a distance d apart, in order that the tape will have its nominal length.

These quantities should be determined as accurately as possible.

To find the tension of accuracy for a tape, place a straight-edge on the point representing the constant AE $\alpha$, axis "A", and on the point representing the observed temperature $t_1$, inclined scale "B". The intersection of the straight-edge with the dummy axis "C" is noted. The straight-edge is then placed on this point and on the point representing the value $P_o$, axis "D". The point where the straight-edge intersects axis "E" is determined. Place the straight-edge on this point and the point representing the constant $\frac{AE\,w^2d^2}{24}$, axis "G". The point where the straight-edge intersects the curve "F" determines the tension to apply to the tape under the given method of support and at the observed temperature $t_1$.

It should be noted that the constants AE$\alpha$, $P_o$, and $\frac{AE\,w^2d^2}{24}$ are constants of the tape, independent of the conditions of its use and can be permanently marked on the chart for each tape, in the office, in preparation for use of the chart in the field.

## IV. Conclusions

A mathematical formula is given with a nomographic chart enabling one to determine readily the tension of accuracy of a

steel tape as used under field conditions.

Experimental verification of the formula is given with information as to the values of the several constants to be used.

# NOMOGRAM FOR STEEL TAPES

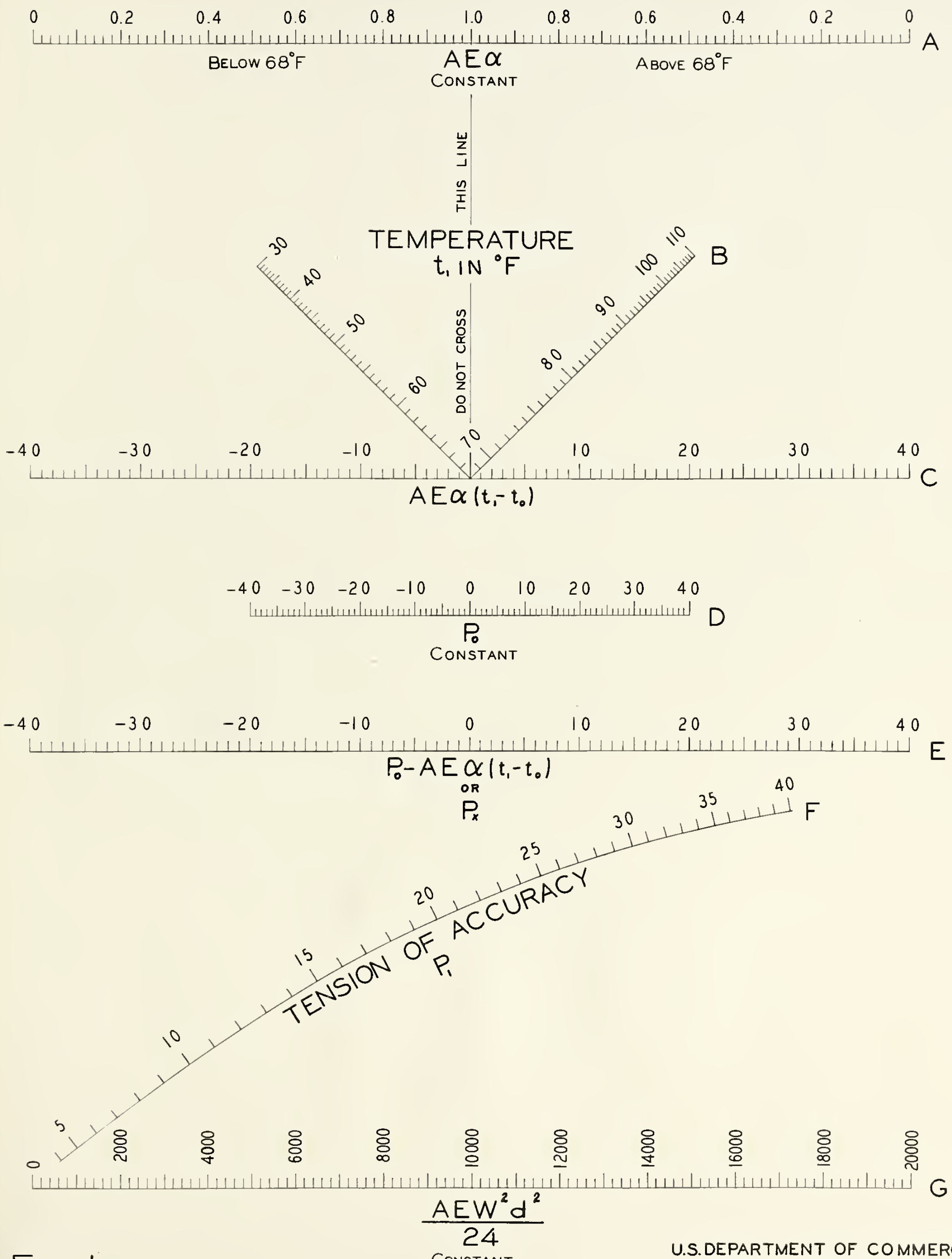


FIG. 1.

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L. C. 313